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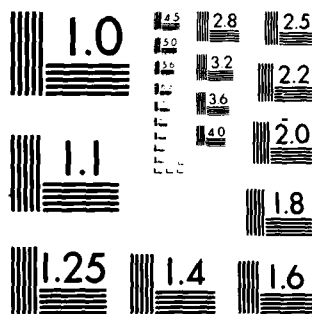
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IN PARTICULATE MEDIA

W.G. Tam

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SPLIT-STEP FOURIER METHOD FOR LASER PULSE PROPAGATION
IN PARTICULATE MEDIA

by

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RESUME

Nous présentons ici une nouvelle méthode pour obtenir la fonction de cohérence mutuelle d'une impulsion laser se propageant au milieu d'aérosols. A partir d'une équation différentielle parabolique, la fonction de cohérence mutuelle est déterminée à l'aide d'une transformation Fourier 'split-step'. Cette approche nous permet d'éviter l'utilisation d'approximations non contrôlées comme ce fut souvent le cas dans les études précédentes. Des algorithmes numériques sous forme de relations algébriques récursives permettent d'étudier le phénomène de l'élargissement de l'impulsion causé par la diffusion multiple. (NC)

ABSTRACT

A new method of obtaining the mutual coherence function of a laser pulse propagating in a particulate medium is described. The treatment, based on the technique of the split-step Fourier transform, enables us to avoid the uncontrolled approximations often used to solve the parabolic differential equation of the mutual coherence function. Simple numerical algorithms in the form of algebraic recurrence relations are derived for the study of the phenomenon of pulse broadening induced by multiple scattering. (U)

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FIGURES 1 to 6

1.0 INTRODUCTION

In the application of electro-optical systems such as laser rangefinders and laser designators, it is important to understand the propagation characteristics of the electromagnetic pulses emitted from the systems. This is particularly true when the pulses have to traverse a turbid medium such as the atmosphere under low-visibility conditions produced by fog, cloud or rain. Apart from energy attenuation, the pulses also suffer distortion which can make the shape of a received pulse significantly different from that emitted. In addition, in atmospheric sensing using a pulsed optical system such as a LIDAR device, the physical information of the medium has to be extracted from the analysis of the characteristics of the returned pulses.

In the study of the pulse propagation characteristics in a medium of randomly distributed scatterers one may consider the evolution of the mutual coherence function as in the work of Beran (Refs. 1, 2) and Ishimaru et al (Refs. 3-5). Alternatively, the technique of temporal moments as considered by Liu and Yeh (Refs. 6-8), or the direct solution of the time dependent radiative transfer equation (Refs. 9, 10) may also be applied.

In the present study, we describe a new method of deriving the mutual coherence function using the split-step Fourier method (Refs. 11, 12). Under the Markov approximation the mutual coherence function satisfies a parabolic differential equation (Ref. 13). This equation formally resembles a time dependent Schroedinger equation with an imaginary 'potential' $iP(\vec{\rho})$ where $P(\vec{\rho})$ is the Fourier transform of the scattering phase function. Under various different approximations, this equation has been solved analytically by Sreenivasiah, Ishimaru, and Hong (Ref. 5) as well as by Zardecki and Tam (Ref. 14). In parallel with these efforts, numerical integrations of the parabolic equations have also been attempted (Refs. 5, 14).

Exploiting a technique similar to the time evolution operator expansion used in solving the Schroedinger equation we derive a recurrence relation for the mutual coherence function with the aid of the split-step Fourier method. This treatment enables us to avoid both the quadratic expansion of $P(\vec{\rho})$ (Ref. 5) or the Glauber approximation in Ref. 14. In other words, the parabolic differential equation for the mutual coherence function can now be solved without invoking simplifying assumptions.

In Section 2.0, the parabolic differential equation of the mutual coherence function is briefly described. The new method of solving this equation by means of the evolution operator expansion and the split-step Fourier scheme is discussed in Section 3.0. In Section 4.0, the problem of stepwise generation of the mutual coherence function is considered. Numerical results for laser pulses propagating in fog are presented in Section 5.0. The work was performed at DREV between December 1979 and July 1980, under PCN 33A11, Aerosol Studies.

2.0 PARABOLIC EQUATION OF THE MUTUAL COHERENCE FUNCTION

A convenient mathematical quantity for the description of the electromagnetic radiation emitted from an extended source is the mutual coherence function (Refs.1, 15). In a random turbulent medium, the propagation of the mutual coherence function has been shown to obey a parabolic equation by Beran (Refs.1, 2). The validity of Beran's results was confirmed by Brown (Ref. 16) solving a Bethe-Salpeter equation of the mutual coherence function. For a medium where the scattering is due to randomly distributed particulate matter, a similar parabolic equation applies provided the scattering events are predominantly in the forward direction (Refs.3-5).

Let $v(\vec{r}, t) \exp(-i\omega_0 t)$ be the output field at a receiver located at \vec{r} produced by an input field $u(t) \exp(-i\omega_0 t)$ at the transmitter located at the origin. The input and output fields are related by the equation (Ref. 17)

$$v(\vec{r}, t) = \int_{-\infty}^{\infty} d\omega U(\omega) H(\omega + \omega_0, \vec{r}, t) \exp(-i\omega t) \quad [1]$$

where $H(\omega, \vec{r}, t)$ is the frequency response function of the medium and $U(\omega)$ is the Fourier transform of $u(t)$. The two-frequency mutual coherence function $\Gamma(\omega_1, \omega_2, \vec{r}_1, \vec{r}_2, t)$ is defined as the ensemble average:

$$\Gamma(\omega_1, \omega_2, \vec{r}_1, \vec{r}_2, t) = \langle H(\omega_1, \vec{r}_1, t) H^*(\omega_2, \vec{r}_2, t) \rangle. \quad [2]$$

The usefulness of the mutual coherence function lies in the fact that given an input signal whose intensity is $I_i(t)$ at the transmitter, the output signal intensity at \vec{r} can be expressed as a convolution

$$I(\vec{r}, t) = \int_{-\infty}^{\infty} dt' G(\vec{r}, t-t') I_i(t') \quad [3]$$

where the Green's function $G(\vec{r}, t)$ is related to $\Gamma(\omega_1, \omega_2, \vec{r}_1, \vec{r}_2, t)$ by the Fourier transform

$$G(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega_d \Gamma(\omega_1 + \omega_0, \omega_2 + \omega_0, \vec{r}, \vec{r}, t) \exp(-i\omega_d t) \quad [4]$$

with $\omega_d = \omega_1 - \omega_2$.

For a plane wave propagating in the z direction, the mutual coherence function at two points $\vec{r}_1 = (\vec{\rho}_1, z)$ and $\vec{r}_2 = (\vec{\rho}_2, z)$ on the same z plane will depend on $\vec{\rho}_d = \vec{\rho}_1 - \vec{\rho}_2$ and z but will be independent of $\vec{\rho}_c = \frac{1}{2}(\vec{\rho}_1 + \vec{\rho}_2)$. Assuming the medium to be time invariant and the scatterings to be uncorrelated and predominantly in the forward direction, it can be shown that (Ref. 17) the mutual coherence function written now as $\Gamma(\omega_d, \vec{\rho}_d, z)$ satisfies the following parabolic equation

$$\left[\frac{\partial}{\partial z} + \frac{ik_d}{2k_o^2} \nabla_d^2 - ik_d + \sigma_t - P(\vec{\rho}_d) \right] \Gamma(\omega_d, \vec{\rho}_d, z) = 0 \quad [5]$$

with the boundary condition

$$\Gamma(\omega_d, \vec{\rho}_d, z = 0) = 1 \quad [6]$$

In eq. 5, $k_o = \omega_o/c$, $k_d = \omega_d/c$, ∇_d^2 is the two-dimensional Laplacian with respect to $\vec{\rho}_d$, σ_t is the volume extinction coefficient and $P(\vec{\rho}_d)$ is the Fourier transform of the scattering phase function $p(\vec{s})$:

$$P(\vec{\rho}_d) = \sigma_s \int d^2\vec{s} p(\vec{s}) \exp(-ik_o \vec{s} \cdot \vec{\rho}_d) \quad [7]$$

with σ_s denoting the volume scattering coefficient. The normalization condition

$$\int d^2 \vec{s} P(\vec{s}) = 1 \quad [8]$$

is assumed.

When the function $P(\vec{\rho})$ is quadratic in $\vec{\rho}$, an analytic solution to eq. 5 is available (Ref. 5). In the more general case, an analytical solution has also been obtained using Glauber's eikonal approximation (Ref. 14). Unfortunately, Glauber's approximation does not reproduce the time asymmetry in the pulse shape introduced by the scattering.

3.0 SPLIT-STEP FOURIER SOLUTION OF THE PARABOLIC EQUATION

Let us introduce a mutual coherence function $\gamma(\omega, \vec{\rho}, z)$ defined by

$$\gamma(\omega, \vec{\rho}, z) = \Gamma(\omega, \vec{\rho}, z) \exp(\sigma_t z - ikz) \quad [9]$$

where we have simplified the notation so that $\omega \equiv \omega_d$, $\vec{\rho} \equiv \vec{\rho}_d$. Equation 5 becomes

$$\frac{\partial}{\partial z} \gamma(\omega, \vec{\rho}, z) = \left[-ia\Delta + P(\vec{\rho}) \right] \gamma(\omega, \vec{\rho}, z) \quad [10]$$

with $\Delta \equiv \nabla_d^2$, $a = k/2k_0^2$ and the boundary condition

$$\gamma(\omega, \vec{\rho}, z = 0) = 1 \quad [11]$$

From eqs. 3-4 the function $\gamma(\omega, \vec{\rho} = 0, z)$ is required in order to evaluate the output pulse $I(\vec{r}, t)$.

Consider the expansion

$$\gamma(\omega, \vec{\rho}, z + \delta z) = \gamma(\omega, \vec{\rho}, z) + \frac{\partial \gamma}{\partial z} \delta z + \frac{1}{2!} \frac{\partial^2 \gamma}{\partial z^2} (\delta z)^2 + \dots \quad [12]$$

By means of eq. 10 one can write

$$\gamma(\omega, \vec{\rho}, z + \delta z) = \exp [\delta z (-ia\Delta + P(\vec{\rho}))] \gamma(\omega, \vec{\rho}, z) \quad [13]$$

For the operator $\exp [\delta z (-ia\Delta + P(\vec{\rho}))]$ we introduce the following approximation:

$$\exp [\delta z (-ia\Delta + P(\vec{\rho}))] = \exp \left[\frac{\delta z}{2} P(\vec{\rho}) \right] \exp [-ia\delta z \Delta] \exp \left[\frac{\delta z}{2} P(\vec{\rho}) \right] \quad [14]$$

The error in equating the two sides of eq. 14 is of the order $(\delta z)^3$.

Let $g(\vec{\rho}) \equiv g(x, y)$ be an arbitrary square integrable function whose Fourier transform is $\hat{g}(\vec{\lambda})$:

$$g(\vec{\rho}) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} d^2\vec{\lambda} \exp(-i\vec{\lambda}\vec{\rho}) \hat{g}(\vec{\lambda}) \equiv F^{-1}[\hat{g}(\vec{\lambda})] \quad [15]$$

Since

$$\Delta^n g(\vec{\rho}) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} d^2\vec{\lambda} (-\vec{\lambda}^2)^n \exp(-i\vec{\lambda} \cdot \vec{\rho}) g(\vec{\lambda}) \quad [16]$$

we have

$$\begin{aligned} \exp(-ia\delta z \Delta) g(\vec{\rho}) &= \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} d^2\vec{\lambda} \exp[ia\delta z \vec{\lambda}^2] \exp(-i\vec{\lambda} \cdot \vec{\rho}) g(\vec{\lambda}) \\ &= F^{-1} [\exp(ia\delta z \vec{\lambda}^2) F[g(\vec{\rho})]] \end{aligned} \quad [17]$$

Combining eqs. 13, 14 and 17 we obtain the following equation

$$\gamma(\omega, \vec{\rho}, z, z+\delta z) = \exp\left(\frac{\delta z}{2} P(\vec{\rho})\right) \left[F^{-1} \left[\exp(ia\delta z \vec{\lambda}^2) F \left[\exp\left(\frac{\delta z}{2} P(\vec{\rho})\right) \gamma(\omega, \vec{\rho}, z) \right] \right] \right] \quad [18]$$

which is the recurrence relation of $\gamma(\omega, \vec{\rho}, z)$ obtained by applying the split-step Fourier algorithm (Refs. 11, 12) to eq. 10.

In the case where the function $P(\vec{\rho})$ possessed cylindrical symmetry about the z axis, the following simplification can be made.

$$\begin{aligned} &F \left[\exp\left(\frac{\delta z}{2} P(\vec{\rho})\right) \gamma(\omega, \vec{\rho}, z) \right] \\ &= 2\pi \int_0^{\infty} d\lambda \lambda \exp\left(\frac{\delta z}{2} P(\rho)\right) \gamma(\omega, \rho, z) J_0(\lambda \rho) \end{aligned} \quad [19]$$

where $J_0(x)$ is the Bessel function of order zero. Equation 18 now becomes

$$\begin{aligned}
 \gamma(\omega, \rho, z + \delta z) &= \exp\left(\frac{\delta z}{2} P(\rho)\right) \int_0^\infty d\lambda \exp(i a \lambda^2 \delta z) \lambda J_0(\lambda \rho) \\
 &\quad \int_0^\infty d\rho' \rho' \exp\left(\frac{\delta z}{2} P(\rho')\right) J_0(\lambda \rho') \gamma(\omega, \rho', z) \\
 &= \exp\left(\frac{\delta z}{2} P(\rho)\right) \int_0^\infty d\rho' \rho' \exp\left(\frac{\delta z}{2} P(\rho')\right) \gamma(\omega, \rho', z) \\
 &\quad \int_0^\infty d\lambda \lambda \exp(i a \lambda^2 \delta z) J_0(\lambda \rho) J_0(\lambda \rho')
 \end{aligned} \tag{20}$$

With the aid of the formulas (Ref. 18)

$$\begin{aligned}
 &\int_0^\infty d\lambda \lambda \left[\begin{pmatrix} \cos \\ \sin \end{pmatrix} (a \lambda^2 \delta z) \right] J_0(\lambda \rho) J_0(\lambda \rho') \\
 &= \frac{1}{2a\delta z} \left[\begin{pmatrix} \sin \\ \cos \end{pmatrix} \left(\frac{\rho^2 + \rho'^2}{4a\delta z} \right) \right] J_0\left(\frac{\rho\rho'}{2a\delta z}\right)
 \end{aligned} \tag{21}$$

Eq. 21 can be reduced to

$$\begin{aligned}
 \gamma(\omega, \rho, z + \delta z) &= \exp\left(\frac{\delta z}{2} P(\rho)\right) \int_0^\infty d\rho' \rho' \exp\left(\frac{\delta z}{2} P(\rho')\right) \gamma(\omega, \rho', z) \\
 &\quad \frac{i}{2a\delta z} \exp\left(-i \frac{\rho^2 + \rho'^2}{4a\delta z}\right) J_0\left(\frac{\rho\rho'}{2a\delta z}\right) \\
 &= \frac{i}{2a\delta z} \int_0^\infty d\rho' \rho' \exp\left[\frac{\delta z}{2} (P(\rho) + P(\rho'))\right] \exp\left(-i \frac{\rho^2 + \rho'^2}{4a\delta z}\right) \\
 &\quad J_0\left(\frac{\rho\rho'}{2a\delta z}\right) \gamma(\omega, \rho, z)
 \end{aligned} \tag{22}$$

In the case of a laser beam passing through fog, cloud or rain the single scattering law can be modelled by a Gaussian scattering phase function (Refs. 19, 20)

$$p(\vec{s}) = \frac{\sigma_s \alpha}{\pi} \exp(-\alpha \vec{\phi}^2) \tag{23}$$

where $\vec{\phi}$ is the projection of \vec{s} on the x-y plane. Corresponding to this, the function $P(\vec{\rho})$ defined by eq. 7 can be written as

$$P(\vec{\rho}) = \sigma_s \exp(-k_0^2 \vec{\rho}^2 / 4\alpha) \tag{24}$$

To simplify our notation let us introduce a length ρ_0 and an angular frequency ω_0 defined by

$$\rho_0 = \left(\frac{4\alpha}{k_0^2}\right)^{1/2}, \quad \omega_0 = 8c\alpha\sigma_s \tag{25}$$

and introduce $\delta\tau_s = \sigma_s \delta z$. In terms of these quantities, we can easily obtain

$$\delta z P(\rho) = \delta z \sigma_s \exp\left(-\frac{k_o^2 \rho^2}{4\alpha}\right) = \delta\tau_s \exp\left(-\frac{\rho^2}{\rho_o^2}\right) = \delta\tau_s \exp(-\bar{\rho}^2) \quad [26]$$

and

$$a\delta z = \left(\frac{k_d}{2k_o^2}\right) \delta z = \frac{k_d}{2} \frac{\rho_o^2}{4\alpha} \delta z = \frac{\omega}{\omega_o} \rho_o^2 \delta\tau_s = \chi \rho_o^2 \delta\tau_s \quad [27]$$

In eqs. 26 and 27 we have set $\bar{\rho} = \rho/\rho_o$ and $\chi = \omega/\omega_o$. When the scattering phase function can be represented by eq. 23, the mutual coherence function $\gamma(\omega, \bar{\rho}, \tau_s)$ will satisfy the equation

$$\begin{aligned} \gamma(\omega, \bar{\rho}, \tau_s + \delta\tau_s) &= \frac{i}{2\chi\delta\tau_s} \int_0^\infty d\bar{\rho}' \bar{\rho}' \exp\left[\frac{\delta\tau_s}{2} \left\{ \exp(-\bar{\rho}^2) + \exp(-\bar{\rho}',^2) \right\}\right] \\ &\quad \exp\left[-\frac{i}{4} \frac{\bar{\rho}^2 + \bar{\rho}',^2}{\chi\delta\tau_s}\right] J_0\left(\frac{\bar{\rho}\bar{\rho}'}{2\chi\delta\tau_s}\right) \gamma(\omega, \bar{\rho}, \tau_s) \\ &= 2i\mu \int_0^\infty d\bar{\rho}' \bar{\rho}' \exp\left[\frac{\delta\tau_s}{2} \left\{ \exp(-\bar{\rho}^2) + \exp(-\bar{\rho}',^2) \right\}\right] \\ &\quad \left[\exp\left[-i\mu (\bar{\rho}^2 + \bar{\rho}',^2)\right] J_0(2\mu\bar{\rho}\bar{\rho}') \gamma(\omega, \bar{\rho}, \tau_s) \right] \end{aligned} \quad [28]$$

with $\mu = (4\chi\delta\tau_s)^{-1}$

Choosing the step size $\delta\tau_s$ to be sufficiently small, one can use the approximation

$$\exp\left[\frac{\delta\tau_s}{2}\left\{\exp(-\bar{\rho}^2) + \exp(-\bar{\rho}',^2)\right\}\right] = 1 + \frac{\delta\tau_s}{2}\left[\exp(-\bar{\rho}^2) + \exp(-\bar{\rho}',^2)\right] \quad [29]$$

in eq. 28 such that

$$\begin{aligned} \gamma(\omega, \bar{\rho}, \tau_s + \delta\tau_s) = 2i\mu \int_0^\infty d\bar{\rho}' \bar{\rho}' \left\{ 1 + \frac{\delta\tau_s}{2} \left[\exp(-\bar{\rho}^2) + \exp(-\bar{\rho}',^2) \right] \right\} \\ \exp\left[-i\mu(\bar{\rho}^2 + \bar{\rho}',^2)\right] J_0(2\mu\bar{\rho}\bar{\rho}') \gamma(\omega, \bar{\rho}, \tau_s) \end{aligned} \quad [30]$$

Equation 30 is the relation which enables us to generate $\gamma(\omega, \bar{\rho}, \tau_s)$ from $\gamma(\omega, \bar{\rho}, \tau_s)$.

At this point, it may be worthwhile to comment on the approximation in eq. 29 in comparison to the quadratic approximation $P(\rho) \sim \sigma_s [1 - k^2 \rho^2 / 4\alpha]$ used in Refs. 5, 17. In eq. 29 it is evident that the error involved is of the order $\delta\tau_s^2$ and this can be controlled by a judicious choice of the step size. The basis of using the quadratic approximation, on the other hand, has not been clarified. In fact, the use of the quadratic approximation in the present treatment will lead to incorrect results.

4.0 STEPWISE GENERATION OF THE MUTUAL COHERENCE FUNCTION

Based on eq. 30 derived in the previous section, a simple algebraic relation can be obtained for the generation of the mutual coherence function. Symbolically we can express eq. 30 as

$$\gamma(\omega, \bar{\rho}, \tau_s + \delta\tau_s) = [G_1 + G_2] \gamma(\omega, \bar{\rho}, \tau_s) \quad [31]$$

in which the integral transformation operators G_1 , G_2 are defined as the following:

$$G_1[\gamma(\omega, \bar{\rho}, \tau_s)] = 2i\mu \int_0^\infty d\bar{\rho}' \bar{\rho}' \exp[-i\mu(\bar{\rho}^2 + \bar{\rho}'^2)] J_0(2\mu\bar{\rho}\bar{\rho}') \gamma(\omega, \bar{\rho}', \tau_s) \quad [32]$$

and

$$G_2[\gamma(\omega, \bar{\rho}, \tau_s)] = 2i\mu \frac{\delta\tau_s}{2} \int_0^\infty d\bar{\rho}' \bar{\rho}' [\exp(-\bar{\rho}^2) + \exp(-\bar{\rho}'^2)] \cdot \exp[-i\mu(\bar{\rho}^2 + \bar{\rho}'^2)] J_0(2\mu\bar{\rho}\bar{\rho}') \gamma(\omega, \bar{\rho}', \tau_s) \quad [33]$$

We note that $G_1\gamma$ and $G_2\gamma$ are of the zeroth and first orders of $\delta\tau_s$ respectively.

Consider the effects of applying the operators G_1 and G_2 to the function $C \exp(-\alpha\bar{\rho}^2)$. It can be seen that

$$\begin{aligned}
 G_1[C \exp(-\alpha \bar{\rho}^2)] &= 2i\mu \int_0^\infty d\bar{\rho}' \bar{\rho}' \exp[-i\mu(\bar{\rho}^2 + \bar{\rho}'^2)] J_0(2\mu \bar{\rho} \bar{\rho}') C \exp(-\alpha \bar{\rho}'^2) \\
 &= \frac{i\mu C}{(\alpha + i\mu)} \exp \left[-\bar{\rho}^2 \left(\frac{\mu^2}{\alpha + i\mu} + i\mu \right) \right]
 \end{aligned}
 \tag{34}$$

Equation 34 can also be expressed as

$$G_1[C \exp(-\alpha \bar{\rho}^2)] = C' \exp(-\alpha' \bar{\rho}^2) \tag{35}$$

where

$$C' = \frac{i\mu C}{\alpha + i\mu}, \quad \alpha' = \frac{\mu^2}{\alpha + i\mu} + i\mu \tag{36}$$

Separating C' and α' into real and imaginary parts such that

$C' = C'_R + iC'_i$ and $\alpha' = \alpha'_R + i\alpha'_i$, one gets

$$C'_R = \frac{\mu[(\alpha_i + \mu)C_R - \alpha_R C_i]}{\alpha_R^2 + (\alpha_i + \mu)^2} \tag{37a}$$

$$C'_i = \frac{\mu[\alpha_R C_R + (\alpha_i + \mu)C_i]}{\alpha_R^2 + (\alpha_i + \mu)^2} \tag{37b}$$

$$\alpha'_r = \frac{\mu^2 \alpha_r}{\alpha_r^2 + (\alpha_i + \mu)^2} \quad [37c]$$

$$\alpha'_r = \mu - \frac{\mu^2 (\alpha_i + \mu)}{\alpha_r^2 + (\alpha_i + \mu)^2} \quad [37d]$$

Similarly, we can write

$$G_2[C \exp(-\alpha \rho^2)] = C'' \exp(-\alpha' \rho^2) \quad [38]$$

where

$$C''_r = \left(\frac{\delta \tau_s}{2} \right) \frac{\mu [(\alpha_i + \mu) C_r - (1 + \alpha_r) C_i]}{(1 + \alpha_r)^2 + (\alpha_i + \mu)^2} \quad [39a]$$

$$C''_i = \left(\frac{\delta \tau_s}{2} \right) \frac{\mu [(1 + \alpha_r) C_r + (\alpha_i + \mu) C_i]}{(1 + \alpha_r)^2 + (\alpha_i + \mu)^2} \quad [39b]$$

$$\alpha''_r = 1 + \frac{\mu^2 (1 + \alpha_r)}{(1 + \alpha_r)^2 + (\alpha_i + \mu)^2} \quad [39c]$$

$$\alpha''_i = \mu - \frac{\mu^2 (\alpha_i + \mu)}{(1 + \alpha_r)^2 + (\alpha_i + \mu)^2} \quad [39d]$$

For an optical path of thickness $\tau_s = n\delta\tau_s$, the mutual coherence function $\gamma(\omega, \bar{\rho}, \tau_s)$ can be generated from the relation

$$\gamma(\omega, \bar{\rho}, \tau_s) = [G_1 + G_2]^n \gamma(\omega, \bar{\rho}, 0) \quad [40]$$

with the boundary condition $\gamma(\omega, \bar{\rho}, 0) = 1$. As the operators G_1, G_2 do not commute, the expansion of $[G_1 + G_2]^n$ gives rise to 2^n monomials in G_1 and G_2 . From eqs. 35 and 38, each of these monomials will contribute a term of the form $C \exp(-\alpha \bar{\rho}^2)$ to $\gamma(\omega, \bar{\rho}, \tau_s)$ and all the intermediate results are of the same functional form. The evaluation of these terms can now be carried out by means of the simple algebraic relations in eqs. 35-39. It may also be noted that in the expansion of $[G_1 + G_2]^n$ there are nC_m terms of the m -th order in $\delta\tau_s$. The point at which the finite series can be terminated in practical computation is effectively controlled by the quantity ${}^nC_m (\delta\tau_s)^m$.

For large values of n the quantity nC_m increases rapidly with m before it attains its maximum value. For example, if $n = 30$, $m = 3$ we have 4060 terms in the expansion of $[G_1 + G_2]^{30}$ which are of the order $(\delta\tau_s)^3$. We have found that an efficient and accurate computational device can be used. This consists of randomly selecting a subset of the set of nC_m terms of the same order in $\delta\tau_s$ for $m \geq 3$, and calculating the mean value of their contributions to $\gamma(\omega, \bar{\rho} = 0, \tau_s)$. The total contribution of all the terms of order $(\delta\tau_s)^m$ is then approximated by nC_m times the mean value of the random set.

5.0 PULSE PROPAGATION IN RADIATIONAL FOG

In this section we apply the previous results to the propagation of a laser pulse with a wavelength equal to $1.06 \mu\text{m}$ in radiational fog. It has been shown (Refs. 19, 20) that for a radiational fog with 0.1 g/m^3 liquid water content, the scattering phase function can be represented by a Gaussian function

$p(\theta) = \frac{\sigma_s \alpha}{\pi} \exp(-\alpha \theta^2)$ with $\alpha = 795.2 \text{ rad}^{-2}$, $\sigma_s = 21.775 \text{ km}^{-1}$ and θ denoting the scattering angle. Neglecting the small absorptance, Figs. 1-3 show the results for the mutual coherence functions $\gamma(\omega, \bar{\rho} = 0, \tau_s)$ with $\tau_s = 2, 4$ and 6 respectively as a function of the frequency $\chi = \omega/\omega_0$. The step size used in $\delta\tau_s$ was 0.2 .

Assuming an input pulse to be of the form

$$I_i(t) = \left(\sqrt{2\pi T} \right)^{-1} \exp(-t^2/2T^2) \quad [41]$$

it can be readily seen that the output pulse is given by

$$I \left(t + \frac{z}{c} \right) = \frac{e^{-\tau_s}}{\pi} \int_0^\infty d\omega \exp \left(-\frac{\omega^2 T^2}{2} \right) [\gamma_R \cos \omega t + \gamma_I \sin \omega t] \quad [42]$$

where γ_R and γ_I are the real and imaginary parts of $\gamma(\omega, \bar{\rho} = 0, \tau_s = \sigma_s z)$. Figures 4-6 show the output pulses for the cases where $T = 20 \text{ ns}$, 2 ns and 0.2 ns with all the pulses normalized in such a way that $\max I(t) = 1$. As one would expect, an increase in the optical depth results in an increase in the pulse distortion. For a given optical depth, on the other hand, the shape distortion in shorter pulses is more severe.

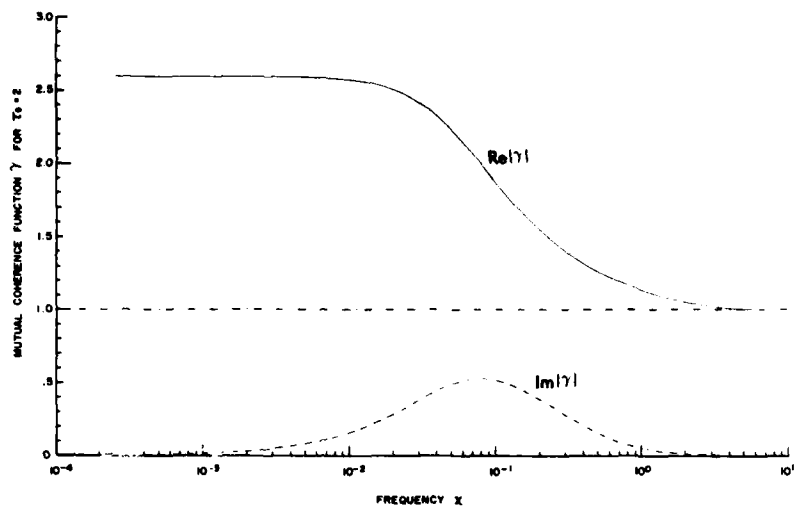


FIGURE 1 - Mutual coherence function γ versus frequency χ for $\tau_s = 2$

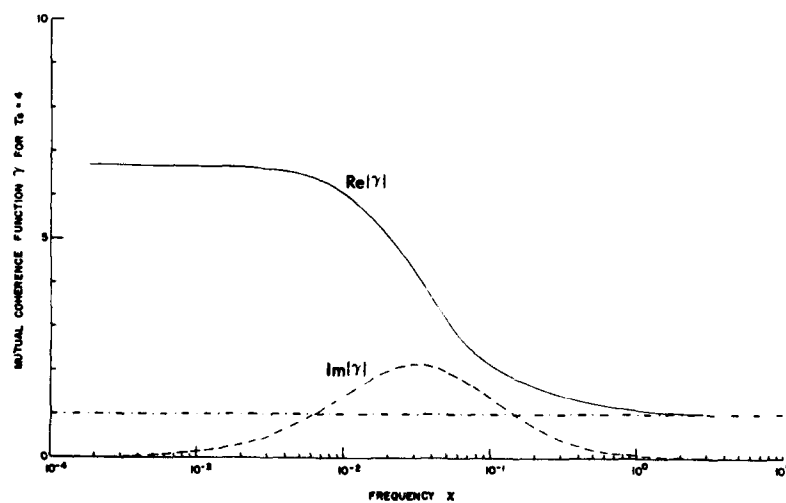


FIGURE 2 - Mutual coherence function γ versus frequency χ for $\tau_s = 4$

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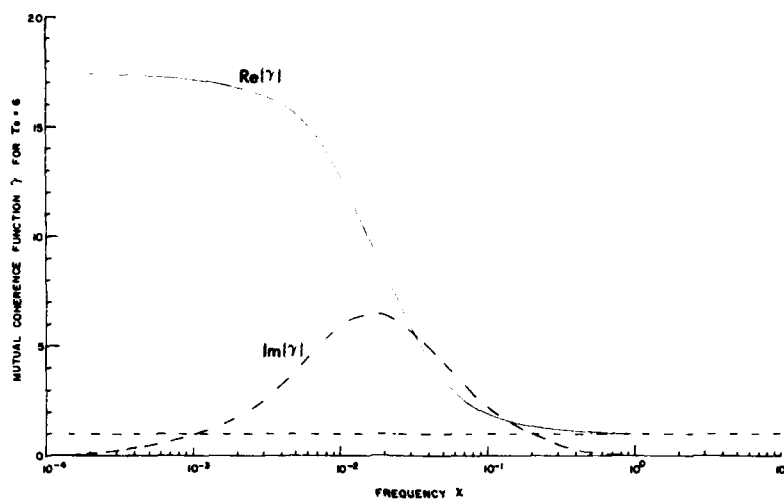


FIGURE 3 - Mutual coherence function γ versus frequency λ for $\tau_s = 6$

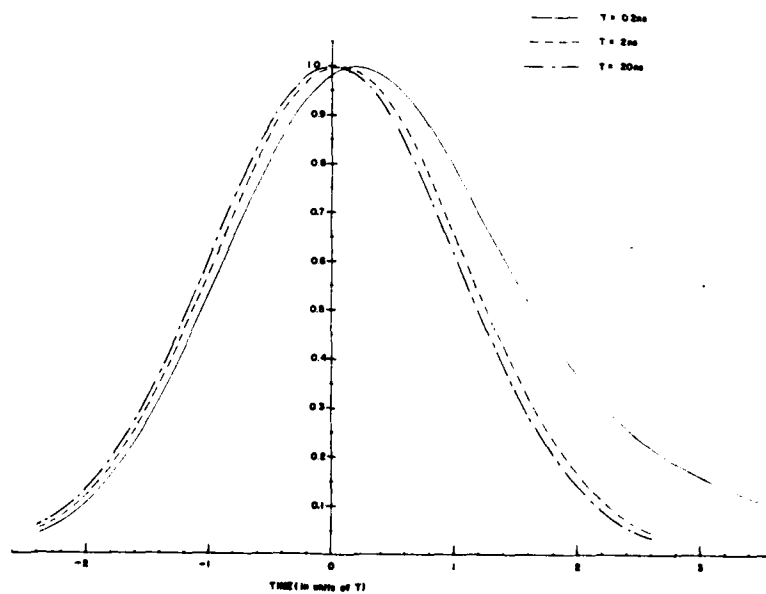


FIGURE 4 - Output pulse intensity for $\tau_s = 2$

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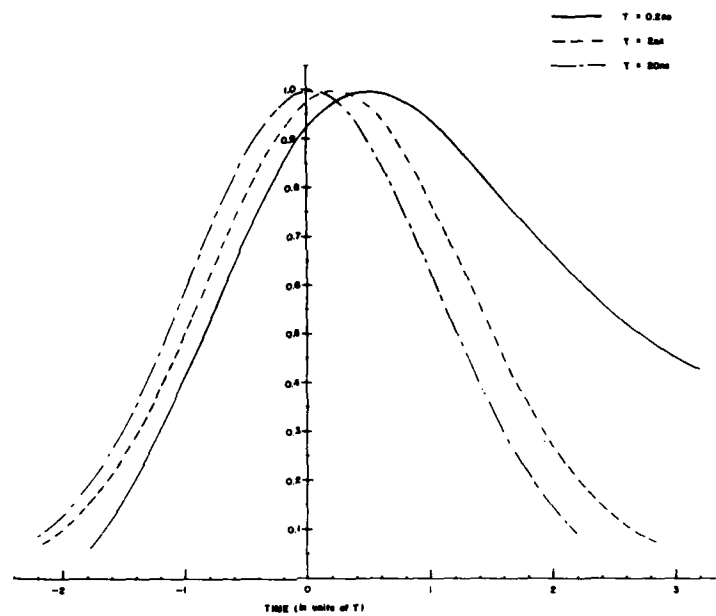


FIGURE 5 - Output pulse intensity for $\tau_s = 4$

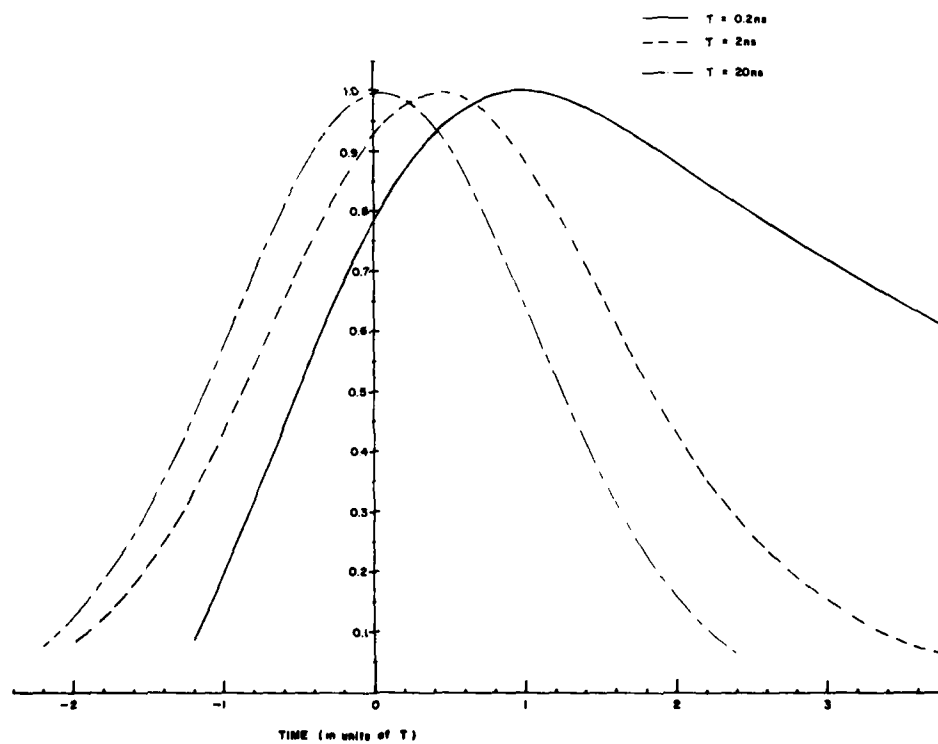


FIGURE 6 - Output pulse intensity for $\tau_s = 6$

6.0 CONCLUSION

We have discussed a new method of solving the parabolic differential equation of the mutual coherence function based on the split-step Fourier method. The principal result is a set of simple algebraic recurrence relations from which the mutual coherence function can be generated for an arbitrary optical depth. Compared to previous work, the present treatment does not require any additional assumption to solve the parabolic equation. The results are applicable to turbid media provided the typical scatterer dimension is comparable or larger than the pulse wavelength.

7.0 ACKNOWLEDGEMENTS

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